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# Analysis of Intrinsic Photoconductors

# Used for Infrared Imaging Purposes

| SYSTEMS RESEARCH AND PLANNING DIVISION | A.S. Gutman |
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AEROSPACE CORPORATION

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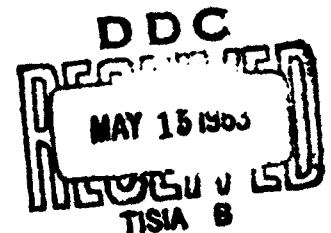
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El Segundo, California

Prepared for

COMMANDER SPACE SYSTEMS DIVISION
UNITED STATES AIR FORCE
Inglewood, California

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# ANALYSIS OF INTRINSIC PHOTOCONDUCTORS
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30 November 1962

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## ABSTRACT

Infrared imaging systems have been proposed in which one photocell can detect many different resolution elements of the image by novel means of scanning. In previous systems one photocell processes the information of one resolution element at any one instant of time. This paper attempts to analyze and evaluate different systems.

Part I -- The discussion begins with an analysis of a photoconductive cell. The reasons for the choice of photoconductive cells for infrared are discussed. The signal-to-noise ratio is derived for weak signals under the assumption that the current noise is predominant. Changes in cell dimensions and illumination patterns are investigated.

Part II - The cell analysis is applied to imaging systems. Sequential scanning systems, mosaic systems, and reticle scanning with orthogonal functions are treated.

# CONCLUSIONS

## Part I - The Photoconductive Cell

It is found that the cell signal-to-noise voltage ratio increases with the modulation frequency as long as this frequency is smaller than the inverse response time. It is inversed proportional to the square root of the area for constant energy of illumination.

## Part II - Imaging Systems Using Photoconductive Cells

The sequential system scanning with one cell over $n$ resolution elements is chosen as the standard comparison system. It is found that the mosaic system requiring $n$ cells results in an improvement of the signal-to-noise voltage ratio by a factor $\sqrt{n}$. A frequency coded system is found to be approaching the performance of the mosaic system in signal-to-noise ratio for detecting point sources. A system using a reticle with an orthogonal binary function can theoretically improve the signal-to-noise voltage ratio by a factor of $\sqrt{2}$. It has, in addition, some background filtering capabilities. Both the frequency coded and the binary function systems require no other moving parts besides the scanning reticle. In addition, they can be used in conjunction with high $f^\#$ number optical systems, giving results equivalent or better than sequential scanning or mosaic systems requiring a lower $f^\#$ number.

## ILLUSTRATIONS

# RECOMMENDATIONS

The reticle scanned infrared imaging systems should be further investigated because they offer the following advantages.

    1.   By relaxing the requirements for low $f\#$ numbers, the difficulties in design of wide angle, high resolution, low $f\#$ number optical systems are avoided.

    2.   The systems have the advantage of few moving parts. (Moving parts present problems in reliability and attitude stabilization of satellites.)

    3.   No slip rings are required.

    4.   Signal-to-noise ratio is improved over presently used sequential scanning systems.

    5.   Some background filtering capability is provided for some systems.

# CONTENTS

CONTENTS (Continued)

# I. THE PHOTOCONDUCTIVE CELL

Some treatments consider photon noise limited detectors under conditions of constant flux densities.

For imaging systems used to detect point-like sources, constant flux is the more fundamental quantity. The magnitude of this flux depends on the energy radiated by the point source and the size of the aperture of the optical system. In addition to this, photoconductors develop a noise power proportional to the square of the current. Such noise seems to be present whenever there is a current-flow. In many practical systems this constitutes the predominant noise.

This paper will demonstrate that practical cells in which current-noise predominates and where the total flux is constant instead of the flux density, have a signal-to-noise voltage ratio inverse to the square root of their area. This is valid for fully and partially illuminated cells.

## A. Reasons for the Use of Photoconductors

Imaging systems based on other phenomena are much more highly developed and work excellently in television and other devices of very high sensitivity. Therefore, a short discussion of these other phenomena and their limitations will be made.

### 1. Photo Emission

One photon causes the release of one electron into vacuum. However, the energy of the photon has to be larger than the work function of the metallic surface from which the electron is emitted.

$$E_{volt} = \frac{hc}{\lambda} = \frac{1.236}{\lambda}$$

when

    c    velocity of light

    h    Planck's quantum

    $\lambda$    wavelength in microns

The metal with the lowest known work function is cesium-silver oxide. $E = 1$ volt; therefore $\lambda = 1.24$ microns is the longest wavelength detectable by this mechanism.

### 2. Photo Voltaic

One photon causes one electron to jump over the potential barrier of a junction of p type and n type solid material. However, this phenomena is not treated in this paper as only the photoconductive effect was considered.

### 3. Conversion Phenomena

The radiation can be converted into heat which can be sensed by a variety of instruments. This conversion-type sensor is not as sensitive as the photoconductor.

### 4. Summary of Reasons for Using Photoconductors

To sense infrared radiation in a wavelength longer than 1.2 micron, the best choice for a highly sensitive detector is the photoconductor unless the wavelength becomes too long, then one must employ an energy conversion device.

## B. The Photoconductive Phenomena

Photoconductivity takes place in certain semiconductors. One can classify these into intrinsic and extrinsic semiconductors.

### 1. Intrinsic Photoconductors

The photon must have enough energy to have one electron jump from the valence band to the conductor band over the forbidden band. The forbidden band is the energy gap which exists between the valence and conduction bands. Materials with a wide forbidden band are dielectrics. The photon energy of infrared radiation is comparatively small and therefore intrinsic photoconductors used for infrared detection are poor insulators. This excludes use of imaging tubes like the vidicon which depend on a storage of electric charge on the surface of the detector.

### 2. Extrinsic Photoconductors

Impurities acting as donors or acceptors of electrons when added to semiconductors can make these semiconductors suitable for detection

of infrared radiation, even in cases where these semiconductors have forbidden bands of wide gaps and are good insulators. However, in these extrinsic semiconductors the total number of free electrons must be small compared to the number of donor electrons. The total number of free electrons is related to the inverse of the absolute temperature in an exponential way. Therefore, all extrinsic semiconductors require low operating temperatures. At the present time the cooling requirements for these detectors cannot be met for space applications where long reliable operating times are required.

### C. Description of Intrinsic Photoconductors

#### 1. General

The intrinsic photoconductors suitable for infrared must have a forbidden band with an energy gap smaller than the infrared photon energy. Therefore, the intrinsic photoconductor cannot be a good insulator because a good insulator requires a wide energy gap in the forbidden band. For present day imaging tubes, for instance the vidicon, one requires photoconductors which are good insulators in order to be able to use electric charge storage on the face of the light sensitive retina. This storage ability results in signal integration process in the detector and thereby in increased signal-to-noise ratios. Infrared imaging devices have to depend on different techniques to achieve high signal-to-noise ratio and/or imaging capabilities.

#### 2. Description of Presently Used Intrinsic Photoconductors for Infrared

Individual uncooled sensing cells use the intrinsic photoconductor usually lead-sulphide, and have no integration or storage capability. If radiation penetrates the sensing cell, electrons are released from the valence band and jump to the conduction band thus creating free electrons and holes and increasing the conductivity of the cell. At the same time, a recombination of holes and electrons takes place. As soon as the generation and recombination of free electrons and holes is in balance, maximum conductivity has been obtained. This is achieved in lead-sulphide in about one millisecond. After the incoming radiation is shut off, it takes about one millisecond to restore

the original dark resistivity of the cell, while the recombination of the electrons and holes takes place. In order to detect this change of resistivity caused by the infrared radiation, one connects a bias voltage across the cell and measures the current. The change in magnitude of the dc current is a measure of the intensity of the infrared radiation illuminating the cell. It is very difficult to accurately measure the magnitude of dc currents. This is due to drifts in dc circuits caused by unwanted resistance and voltage variations. A change in current can be measured much more easily. If the change in current is periodic, one has an ac component which can easily be amplified a million times in a tuned amplifier. This is the basic reason why one uses a light chopper in conjunction with photoconductive cells. The light chopper will cause a modulation in the output of the cell. The ac frequency of the current is equal to the chopping frequency. Of course the upper limit of this chopping frequency is given by the inverse of response of the cell. With $\tau$ = time constant for response of cell in seconds, $f \lesssim 1/\tau$, where f frequency in cycles/second.

This chopping practice is so common with infrared detectors that the sensitivity of the detector is measured at some given or optimum chopping frequency. Usually a value $D^*$ is given for a given wavelength or blackbody temperature and a given chopping frequency and a given bandwidth or detection time. The units of $D^*$ are cm, cps$^{1/2}$, watt$^{-1}$ $D^*$ is an inverse radiation intensity. $D^*$ is a parameter of an infrared detection cell measured in a way which presupposes a given detection system because chopping frequency is associated with the $D^*$ measurement. If entirely different detection systems are proposed one must be very careful to extrapolate from presently known values of $D^*$ the performance of systems which use entirely different chopping methods and have different bandwidth requirements. Those extrapolations may have to be confirmed by a new measurement of sensitivity or detectivity $D^*$ under the new system conditions.

### D.   The Photoconductive Cell as a Circuit Element



Dark Resistivity $\rho$

Length $\ell_o$
Width $w_o$
Depth $u$
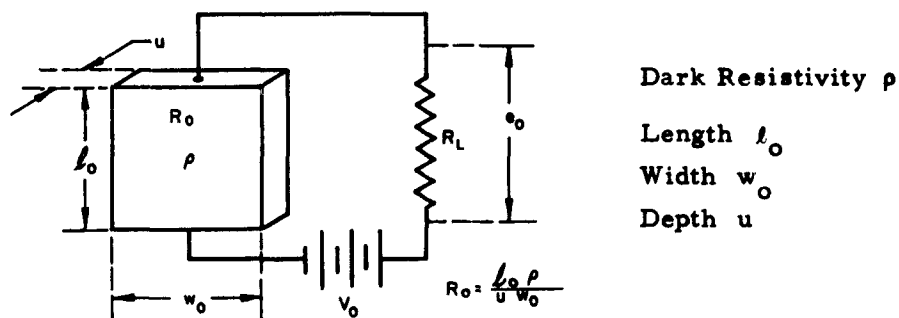
Figure 1.   The Standard Cell.

### 1.   Signal-Voltage

The light is chopped with a frequency $f$.  The alternating illumination causes an alternating resistivity of the cell.  The alternating resistivity causes an alternating current.  The alternating current causes an alternating signal-voltage across the load resistor $R_L$.

$$S_o = R_L \, \delta I \tag{1}$$

where

$\quad S_o \quad$ alternating signal volt

$\quad R_L \quad$ load resistor

$\quad \delta I \quad$ current variation

With the constant bias-voltage: $V_o$

Current:

$$I = \frac{V_o}{R_o + R_L} \tag{2}$$

where

$\quad R_o \quad$ cell resistance

and

$$\delta I = V_o \frac{-\delta R_o}{(R_o + R_L)^2} \qquad (2a)$$

inserted into Equation (1)

$$S_o = -R_L V_o \frac{\delta R_o}{(R_o + R_L)^2} \qquad (3)$$

We wish to find the load resistor $R_L$ which gives maximum signal voltage $S_o$.

$$\frac{dS_o}{dR_L} = V_o \delta R_o \left[ \frac{2 R_L}{(R_o + R_L)^3} - \frac{1}{(R_o + R_L)^2} \right] = 0$$

from which follows:

$$2 R_L = R_o + R_L$$

$$R_L = R_o \qquad (4)$$

Maximum signal voltage is obtained with a load resistor matched to cell resistance. Combining Equations (3) and (4) we get:

$$S_o = - \frac{V_o}{4} \frac{\delta R_o}{R_o} \qquad (5)$$

Note: This result is valid for circuits without reactance. With an ac output circuit reactance should be taken into account in matching the cell to the load. However, this will not change the analysis of the cell behavior.

2. The Noise Voltage[*]

The noise is composed of:

Johnson noise which is proportional to resistance

Radiation noise which is proportional to illuminated area

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[*]Proceedings of the IRE Vol. 47, No. 9, pages 1481-1486, "Noise in Radiation Detectors" by R.C. Jones.

Carrier generation and recombination noise: $N_G$

Modulation noise: $N_M$

In most practical applications of photoconductive cells the predominant internal noise is due to carrier generation and recombination noise and to modulation noise. Therefore, the noise is

$$N_o \approx \sqrt{N_M^2 + N_G^2} \qquad (6)$$

Modulation noise current fluctuation (RMS) is:

$$I_M = \int \frac{K_M^2 \, I'^2}{f} \, df \qquad (7)$$

where

    $K_M$    proportionality constant

    f       modulation frequency

    I'     current under illumination

$$I' = I + \delta i$$

    I       dc current

and for small signals:

$$I' \approx I \qquad (8)$$

For white noise we have from Equation (7) and (8)

$$I_M = \sqrt{K_M^2 \, I^2 \, \frac{\Delta f}{f}} \qquad (9)$$

where $\Delta f$ is the bandwidth. The noise voltage is:

$$N_M = I_M R_o$$

inserted into Equation (9)

$$N_M = K_M I R_o \sqrt{\frac{\Delta f}{f}}$$

and with

$$IR_o = V_o \tag{10}$$

$$N_M = K_M V_o \sqrt{\frac{\Delta f}{f}} \tag{11}$$

Noise current fluctuation due to generation and recombination of carriers is

$$I_G = \int \frac{4I'^2 \tau}{n_o \left[1 + (2\pi f \tau)^2\right]} \, df \tag{12}$$

where

$\tau$    lifetime of carrier

$n_o$    mean number of carriers

Assuming white noise and combining Equations (8), (10) and (12) we obtain the noise voltage.

$$N_G = V_o \sqrt{\frac{4\tau \Delta f}{n_o \left[1 + (2\pi f \tau)^2\right]}} \tag{13}$$

Combining Equations (6), (11) and (13) we get

$$N_o = V_o \sqrt{\Delta f \, K(f)} \tag{14}$$

where

$$K(f) = \frac{K_M^2}{f} + \frac{4\tau}{n_o \left[1 + (2\pi f \tau)^2\right]} \tag{15}$$

Combining Equations (5) and (14) we get

$$\frac{S_o}{N_o} = -\frac{1}{4} \frac{\delta R_o}{R_o} \frac{1}{\sqrt{\Delta f \, K(f)}} \tag{16}$$

We can easily show that

$$\frac{\delta R_o}{R_o} = \frac{\delta \rho_o}{\rho_o}$$

where

$\rho$    resistivity of the photoconductor.

Therefore

$$\frac{S_o}{N_o} = -\frac{1}{4}\frac{\delta\rho_o}{\rho_o}\frac{1}{\sqrt{\Delta f\,K(f)}}\qquad(17)$$

The signal-to-noise voltage ratio of the standard cell depends on

$\delta\rho_o$    change of resistivity with illumination

$\rho_o$    cell resistitivity

$\Delta f$    bandwidth

$K(f)$    a constant dependent on chopping frequency according to Equation (15)

E.    The Variation of Cell Dimensions

Section D.1. discussed the signal voltage as caused by variation in illumination.

Section D.2. discussed the noise voltage of the standard cell.

This section concerns itself with the influence of cell area on signal and noise voltages.

One way to investigate the influence of cell area on signal-to-noise ratio is to analyze what happens when many standard cells are interconnected electrically. Define

$A_o$    area of standard cell

$R_o$    resistance of standard cell

$S_o$    signal voltage of standard cell

$N_o$    noise voltage of standard cell

The analysis is simplified by considering series and parallel connection separately.

*   Two separate cases are treated: All cells equally illuminated; Only one of the cells is illuminated.

1.  **Series Connection of n Equal Cells Equally Illuminated**

    | | |
    |---|---|
    | Resistance of n cells | $R_1 = n R_o$ |
    | Area of n cells | $A_1 = n A_o$ |
    | Signal volt of n cells | $S_1 = n S_o$ |
    | Noise volt of n cells | $N_1 = \sqrt{n} (N_o)$ |

Note: The signal is coherent and adds algebraically. The noise is incoherent and adds with the root mean square.

Signal-to-noise voltage ratio

$$\frac{S_1}{N_1} = \sqrt{n} \left( \frac{S_o}{N_o} \right)$$

substitute $n = A_1 / A_o$

$$\frac{S_1}{N_1} = \sqrt{\frac{A_1}{A_o}} \left( \frac{S_o}{N_o} \right) \tag{18}$$

2.  **Parallel Connection of n Equal Cells Equally Illuminated**

    | | |
    |---|---|
    | Resistance of n cells | $R_2 = 1/n \, R_o$ |
    | Area of n cells | $A_2 = n A_o$ |
    | Signal volt of n cells | $S_2 = S_o$ |
    | Noise volt of n cells | $N_2 = (1/\sqrt{n}) \, N_o$ |

Note: The signal of equal parallel connected voltages is unchanged. The noise fluctuation of incoherent parallel connected noise voltages is reduced with the root mean square.

Signal-to-noise voltage ratio

$$\frac{S_2}{N_2} = \sqrt{n}\left(\frac{S_o}{N_o}\right)$$

substitute $n = A_2/A_o$

$$\frac{S_2}{N_2} = \sqrt{\frac{A_2}{A_o}}\left(\frac{S_o}{N_o}\right)$$

which is equivalent to Equation (18) above.

3. Series Connection of n Equal Cells
   Only One Cell Illuminated

|  |  |
|---|---|
| Resistance of n cells | $R_3 = n\,R_o$ |
| Area of n cells | $A_3 = n\,A_o$ |
| Signal volt of n cells | $S_3 = S_o$ |
| Noise volt of n cells | $N_3 = \sqrt{n}\,N_o$ (incoherent) |

Signal-to-noise voltage ratio

$$\frac{S_3}{N_3} = \frac{1}{\sqrt{n}}\left(\frac{S_o}{N_o}\right)$$

substitute $n = A_3/A_o$

$$\frac{S_3}{N_3} = \sqrt{\frac{A_o}{A_3}}\left(\frac{S_o}{N_o}\right) \tag{19}$$

4. Parallel Connection of n Equal Cells
   Only One Cell Illuminated

|  |  |
|---|---|
| Resistance of n cells | $R_4 = (1/n)\,R_o$ |
| Area of n cells | $A_4 = n\,A_o$ |
| Signal volt of n cells | $S_4 = (1/n)\,S_o$ |
| Noise volt of n cells | $N_4 = (1/\sqrt{n})\,N_o$ |

Note: The current variation of one cell is detected by a load resistor of $(1/n)\,R_o$. Therefore the signal voltage is reduced by $1/n$.

Signal-to-noise voltage ratio

$$\frac{S_4}{N_4} = \frac{1}{\sqrt{n}}\left(\frac{S_o}{N_o}\right)$$

substitute $n = A_4/A_o$

$$\frac{S_4}{N_4} = \sqrt{\frac{A_o}{A_4}}\left(\frac{S_o}{N_o}\right)$$

This is equivalent to Equation (19) above

### F. Imaging of Point Sources

The energy admitted to the optical system by a point source is

$$E = \frac{\pi}{4}\frac{D^2}{R^2} J \tag{20}$$

where

D   diameter of the aperture (in.)

R   distance of the source from the aperture (in.)

J   energy emitted by the source in (watts/steradians)

This energy is concentrated on the sensing cell. Assuming that the optical system distributes this energy on the area $A_o$ of the standard cell, we have a flux-density

$$F_o = \frac{E}{A_o} \tag{21}$$

The change in resistivity is proportional to the flux density

$$\delta\rho \propto F$$

Combined with Equation (21)

$$\delta\rho \propto \frac{E}{A_o} \tag{22}$$

Increasing cell area $A_o$ we have to consider two different operation conditions

      (1)  The illumination is left unchanged.

      (2)  The illumination is redistributed into the increased area.

### 1. Cell Area Increased, Optical Resolution Unchanged

This case has been treated in Section E.3 and 4. The resulting Equation (19) applies. Note energy $E$, flux density $F$ and change of resistance $\delta\rho_o$ remains unchanged in this case.

### 2. Cell Area Increased, Optical Resolution to Match New Cell Area

With the energy admitted to the optical system remaining unchanged the flux density is changed according to Equation (21)

$$F_2 = F_o \frac{A_o}{A_2}$$

combined with Equation (22)

$$\delta\rho_2 = \delta\rho_o \frac{A_o}{A_2} \tag{23}$$

Equation (18) treated the case of $n$ cells equally illuminated, Equation (17) treats the influence of $\delta\rho$ on signal-to-noise ratio. Combining Equations (17) and (18) we get

$$\frac{S_2}{N_2} = \sqrt{\frac{A_1}{A_o}} \left(\frac{\delta\rho_2}{\delta\rho_o}\right)\left(\frac{S_o}{N_o}\right)$$

combined with Equation (23)

$$\frac{S_2}{N_2} = \sqrt{\frac{A_o}{A_2}} \left(\frac{S_o}{N_o}\right)$$

which is equivalent to Equation (19)

### 3. Summary of Influence of Cell Area for Imaging Point Sources

The signal-to-noise voltage ratio is inverse proportional to the square root of the cell area regardless whether the cell is only partially or fully illuminated.

## II. IMAGING SYSTEMS

### A. Standard Infrared Detection System

Before we embark on the analysis of exotic detection systems we elect to establish a standard reference system for the sake of comparison purposes. We choose for the standard reference system a sequential scanning system.

#### 1. Basic Definitions

$n$    resolution elements per frame

$t$    frame time (seconds)

#### 2. Sequential Scanning with One Cell

Integration time available - $t_o = t/n$

Integration is performed after detection in a separate electronic device such as, for example, a matched filter.

#### 3. Assumption

Chopping period is small compared to frame time: $1/f \ll t$. The signal is integrated over the time $t_o$; the noise is also integrated over the same time $t_o$. Therefore the signal is

$$S_o^* = \frac{t}{n} S_o$$

the noise is

$$N_o^* = \sqrt{\frac{t}{n}} \, N_o'$$

The asterisk denotes imaging systems as opposed to photocells. The signal-to-noise voltage ratio is

$$\frac{S_o^*}{N_o^*} = \sqrt{\frac{t}{n}} \frac{S_o}{N_o'} \tag{24}$$

We shall compare all other systems with the signal-to-noise ratio of this reference system.

B. **Mosaic Detection System**

1. **Basic Definitions**

   n    resolution elements

   t    frame time

   n photocells with n separate amplifiers and detectors. Integration time available $t_1 = t$.

2. **Assumption**

   $1/f \ll t$

   With the new integration time $t_1$ the signal-to-noise ratio of this system will be

$$\frac{S_1}{N_1} = \sqrt{t} \; \frac{S_o}{N'_o} \tag{25}$$

3. **Conclusions**

   The mosaic system has an improvement factor $\sqrt{n}$ over the sequential (reference) system.

4. **Remarks**

   For high resolution systems the size, weight and power consumption of the n detection systems may be objectionable.

C. **Reticle Coded Detection Systems**

1. **Basic Definitions**

   n    resolution elements

   t    frame time

2. **Description of System**

   Each resolution element is modulated differently by a reticle. After modulation, the light of all the resolution elements is collected into the sensitive surface of one photocell. The modulated output of this cell is amplified and the output of this amplifier is correlated with the reticle functions so as to separate the signals of the one-photocell into n channels.

### 3. Goals of a Reticle Coded Detection System

The goal of this system is to use one detection cell for n resolution elements. Instead of scanning in a time sequence over the n resolution elements, the desire is to view all n elements simultaneously coding each resolution element with a characteristic coding which makes it possible to separate later the n signals again into n channels. Several methods of reticle coding will be discussed.

### 4. Use of Orthogonal Functions

The orthogonality condition for the chopping function F is

Time Functions

$$\int_0^t F_i(t) \, F_\alpha(t) \, dt < N_3$$

$$\int_0^t F_i(t) \, F_i(t) \, dt > N_3$$

or using

Frequency Functions

$$\int_0^t F_i(f,t) \, F_\alpha(f,t) \, dt < N_3$$

$$\int_0^t F_i(f,t) \, F_i(f,t) \, dt > N_3 \tag{26}$$

where

$N_3$   noise in the system

t   frame time

f   chopping frequency

### 5. Orthogonality of Time Functions

The orthogonality of these functions can be achieved by some type of time sharing. An example of orthogonal functions is shown.
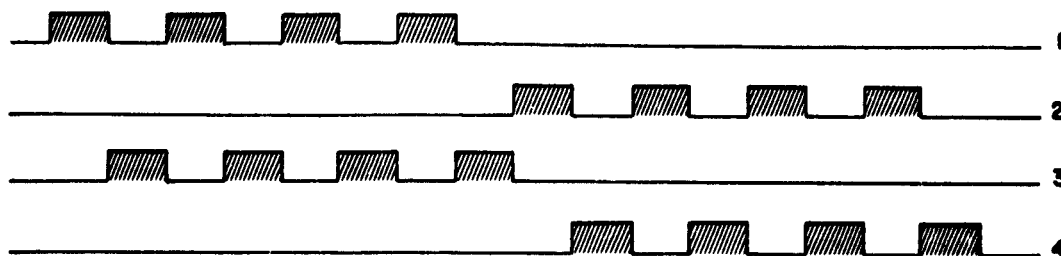
Figure 2.  Orthogonal Function.

A moving belt with openings in place of the shaded rectangles could be used as such an orthogonal function chopper.  Note the interlacing of the channels; therefore the integration time available for each channel would be 2(t/n), twice longer than that in a sequential system using an ordinary light chopper which cuts off the available light half the time.

The coordinates of one point source in the two-dimensional image area are obtained by correlating the signal output with a reticle of same design synchronized with the chopper reticle in the image plane (see Figure 3).  The imaging process can best be visualized if we assume one photocell collecting all the light behind the moving belt reticle and this photo-cell controlling, by means of an amplifier, a point-like light source.  If we move the duplicate reticle synchronously with the chopping reticle over this light source, the light will appear in the resolution element corresponding to the one in which the photocell was illuminated.  The timing of the light flashes will determine at which location of the moving reticle it will show through.

Extended sources of background light would not show up as a signal in this system because the sensor output detects only changes in illumination.  The chopping areas in line 1 and 3 interlace and, therefore, there would be no changes in illumination for light sources which extend over area 1 and 3 simultaneously.  Therefore, the reticle will accomplish a certain amount of background filtering.
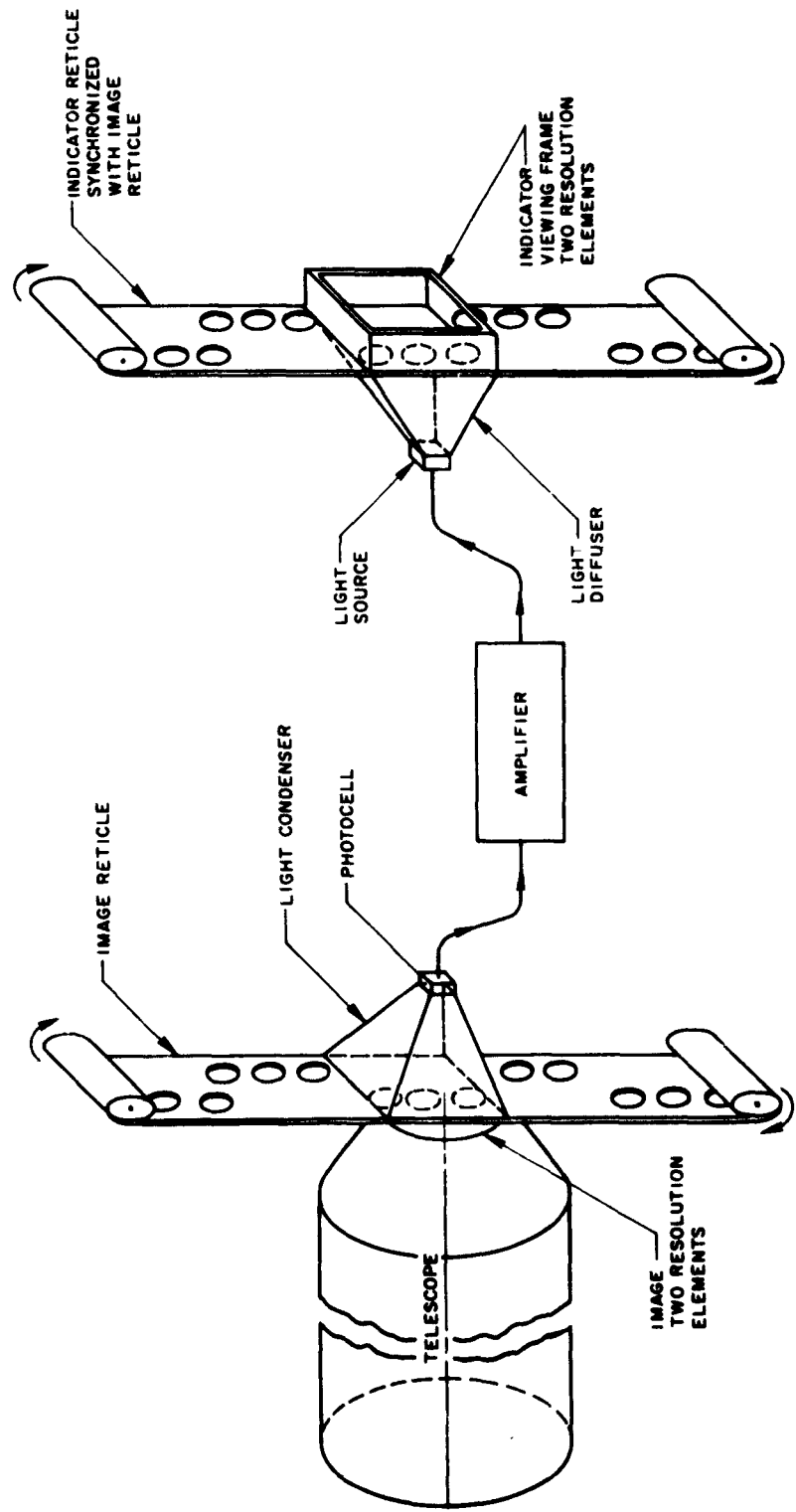
Figure 3.  Example of Reticle Scanning for Two Resolution Elements
Using Optical Correlation in Indicator.

The cell area should not be larger than the cell area of the standard system because the signal-to-noise ratio of the cell is inverse to the square root of the area, as shown in Part I of this paper. In order to accomplish this, one must collect all light in the image area after it has passed through the chopping reticle and optically condense this light on the area of a small cell. There are, however, certain physical limitations connected with this light condensing requirement.

### 6. The Optical Limitations of Light Condensers (Figure 4)

The optical system consists of a lens of focal length $\varphi$ and diameter D which images an object into the images plane. The object may be a square of length $a_2$ and absolute temperature $T_2$ and the image another square of length $a_1$ and temperature $T_1$. The light of the image is chopped by a reticle (not shown in the illustration) and thereafter collected by an optical waveguide shown as a cone into the cell area (a square of length $a_o$ and temperature to $T_o$).



OBJECT-SPACE          IMAGE-SPACE
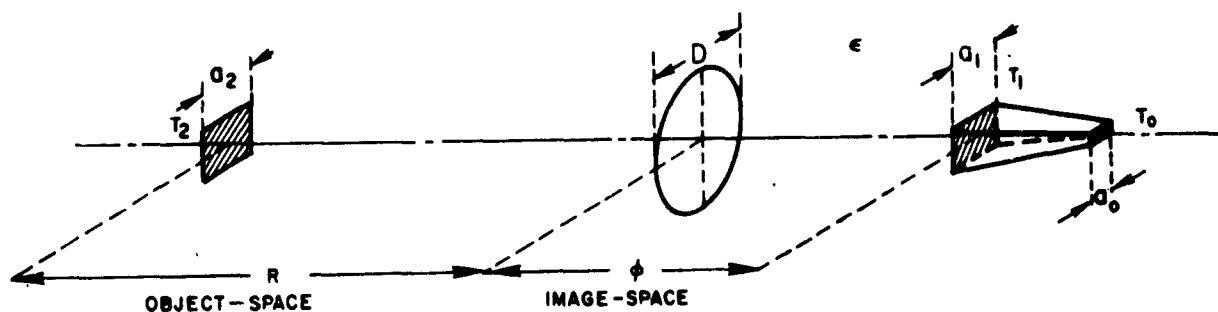
Figure 4. Optical System.

The refractive index in the object space is 1 and the refractive index in the image space is $\epsilon$. For large R the relation between object and image size is

$$\frac{a_2}{a_1} = \frac{R}{\epsilon \varphi} \tag{27}$$

According to the Stefan-Boltzman law the radiation density from the object is

$$F_2 = \sigma T_2^4$$

The energy radiated from the object is

$$E_2 = \sigma T_2^4 \, a_2^2$$

The energy impinging on the aperture is (see Figure 4)

$$E_o = \frac{E_2 \frac{\pi}{4} D^2}{\pi R^2} = \frac{\sigma T_2^4 \, a_2^2 \, D^2}{4 R^2}$$

The imaging Equation (27) is

$$\frac{a_2}{R} = \frac{a_1}{\epsilon \varphi}$$

Combining the last two equations one gets

$$E_o = \sigma T_2^4 \, \frac{a_1^2 \, D^2}{4 \epsilon^2 \varphi^2}$$

The flux density at the image is

$$F_2 = \frac{E_o}{a_1^2} = \sigma T_2^4 \, \frac{D^2}{4 \epsilon^2 \varphi^2}$$

The flux emitted from the image is

$$F_1 = \sigma T_1^4$$

In the equilibrium condition

$$F_1 = F_2$$

and therefore

$$\frac{T_1^4}{T_2^4} = \frac{D^2}{4 \epsilon^2 \varphi^2}$$

or

$$\left(\frac{T_1}{T_2}\right)^2 = \frac{D}{2\epsilon\varphi}$$

according to the second law of thermodynamics

$$T_2 > T_1$$

and therefore

$$\frac{\epsilon\varphi}{D} > \frac{1}{2} \tag{28}$$

This means it is impossible to construct an optical system with an f# number smaller than 1/2.

The same reasoning can be applied to the light condenser from $a_1$ to $a_2$ (see Figure 4).

Therefore, after chopping it is not possible to condense the light further to a small photocell area unless one starts with an optical system with an f# number larger than 1/2 so that the over-all effective sensitivity of this system would be equivalent to the standard system described in Section I-A, with the exception of the gain of $\sqrt{2}$ in integration time discussed in Section II-C5.

### 7. The Use of Orthogonal Frequency Functions

The use of orthogonal frequency functions has the advantage that one gains on integration time a factor n over the sequential scanning system chosen as a reference. This gain in integration time results in a gain by a factor of $\sqrt{n}$ in signal-to-noise ratio.

The use of chopping frequencies other than the optimum chopping frequency, however, results in a loss of signal-to-noise ratio according to Equation (17) in Part I of this report.

Using a matched filter to separate the different chopping frequencies the following condition has to be satisfied

$$f_2 < f_1 - \frac{1}{t}$$

t    frame time

or with  n  resolution elements and therefore  n  frequencies

$$f_n < f_1 - \frac{n-1}{t}$$

or

$$\frac{f_n}{f_1} < 1 - \frac{n-1}{f_1 t} \tag{29}$$

Equations (15) and (16) in Part I of this report show the influence of chopping frequency on signal-to-noise ratio.

If we choose

$$f_1 = \frac{1}{\tau}$$

Where  $\tau$  is the cell response time we get from Equation (15)

$$K(f)_1 = \tau \left\{ K_M^2 + \frac{4}{n_o \left[ 1 + (2\pi)^2 \right]} \right\} \tag{30}$$

and from Equation (29) with  $f_1 = (1/\tau)$

$$f_2 = \frac{1}{\tau} \left( 1 - n\frac{\tau}{t} \right) \tag{31}$$

Combining Equations (31) and (15) we get

$$K(f)_2 = \tau \left\{ \frac{K_M^2}{(1 - \frac{\tau}{t} n)} + \frac{4}{n_o \left[ 1 + 4\pi^2 (1 - \frac{\tau}{t} n)^2 \right]} \right\} \tag{32}$$

Comparing Equations (30) and (32) we see that for

$$\frac{\tau}{t} n < 1 \tag{33}$$

we have

$$K(f)_1 \approx K(f)_2$$

Therefore Equation (33) is a necessary operating condition for the use of frequency coded systems.

8. **Discussion of the Result and Comparison with Sequential Scanning System for 100 Resolution Elements**

Due to the fact that the system has a gain in integration time by a factor of 100, the signal-to-noise voltage ratio of the first channel has a gain of $\sqrt{100} = 10$ and the signal-to-noise voltage ratio of the 100th channel has almost the same gain provided the following conditions are fulfilled.

a. **Frame Time, t**

According to Equation 30.

$$\frac{\tau}{t} n < 1$$

Therefore $t > \tau n$, with lead-sulphide $\tau = 1/1000$ (sec). Therefore for $n = 100$ resolution elements, $\tau n = 100/1000 = 1/10$ which is small compared to 1. Therefore 100 resolution elements can be multiplexed on lead-sulphide detectors with frequency coding, provided the frame time $t \geq 1$ second.

b. **Cell Area**

Due to the effects of cell area on signal-to-noise ratio presented in Part 1 of this report, it is not advisable to use increased cell areas with the frequency coded system. One requires therefore, a light condenser.

c. **f# Number**

The f# number of the frequency coded system has to be high enough to make the use of an optical light condenser possible (see Section II-C6). For general imaging systems the f# number of both systems (the frequency coded system and the sequential scanning system) have to be identical for comparison purposes. If the f# numbers of both systems are approaching theoretical low limits then the sequential scanning

system can operate with a lower f#/ number than the frequency coded system which requires a light condenser after the image plane in order to operate both systems with the same photocell area. In such cases, one must distinguish signal-to-noise ratio for extended illuminated objects or for point-like sources.

(1) For extended objects the higher f# number of the frequency coded system cancels the improvement over the sequential system which is due to the longer integration time available.

(2) For point-like sources at large distances the f# number is not important, but only the size of the aperture determines the energy illuminating the photocell. Therefore, for detecting point-like sources at large distances (large compared to focal-length) the calculated improvement factors are valid.

d. Filtering

Results calculated are based on perfect filtering between different frequency channels. With such filters the bandwidth after the filtering is not substantially different than the bandwidth required in the sequential scanning system. While bandwidth and therefore noise before the filter is larger than in the sequential scanning system, one has no difference in noise per channel after the filtering.

9. Note on Filtering Technique

An effective filtering technique is possible by using resonant electric circuits with a high Q-factor (low dissipation). It is also possible to employ resonant mechanically-vibrating elements such as strings, crystals, etc., which are usually electromagnetically excited. All these frequency filters have resonant structures and the ability to accept and integrate positive and negative amplitude values. Simple optical choppers or reticles combined with an unbiased glow lamp have only positive values of light and therefore cannot perform this filtering action with the same effectiveness. The achievement of equally effective matched filters by optical means may be possible with a system having integration capabilities over positive and negative amplitude values.

## 10. Note on Frequency Coding

The analysis presented uses one fixed frequency for each resolution element. It results in different signal-to-noise ratio for different resolution elements.

## 11. Summary of Detection Systems

The best imaging system is a mosaic system. In cases when it is not possible to use a mosaic system because it requires too high a number of photocells and amplifiers, the standard sequential scanning system can be used at a sacrifice of signal-to-noise ratio. Other systems with the same number of cells as the sequential scanning system can achieve a significant gain in signal-to-noise ratio over the sequential scanning system. A frequency coded system combined with the proper filtering can achieve a significant improvement over a sequential scanning system in signal-to-noise ratio for point-like sources. A reticle system which could be called a modified sequential system has been described which may improve the signal-to-noise ratio by a factor of $\sqrt{2}$ over the regular sequential system. A further advantage of these systems is the elimination of moving parts except for the reticle. In addition, one can achieve good performance with optical systems with higher f # number. Because of the difficulties of designing wide angle, high resolution optical systems with low f # number, this can be an important advantage.

## GLOSSARY

| | |
|---|---|
| A | Cell Area |
| E | Energy of Illumination |
| $R_L$ | Load Resistor |
| V | Cell Bias |
| I | Cell Dark Current |
| I' | Cell Current Under Illumination $(I' \approx I + \delta I)$ |
| S | AC Signal Volt - $S = R_L \delta I$ |
| N | Noise Volt - $N \propto I R_o$ |
| $N_G$ | Carrier Generation and Recombination Noise |
| $N_M$ | Modulation Noise |
| $K_M$ | Modulation Noise Constant |
| e | Output Volt |
| $\rho$ | Dark Resistivity |
| $l$ | Cell Length |
| W | Cell Width |
| u | Cell Thickness |
| f | Chopping Frequency |
| $\tau$ | Response Time of Cell |
| $n_o$ | Mean Number of Carrier |
| $\delta\rho$ | Variation of Resistivity |
| $\delta I$ | Variation of Current |
| $\delta R$ | Variation of Resistance |

|  | Signal Volt | Noise Volt | Integration Time |
|---|---|---|---|
| Sequential Scanning System | $S_o^*$ | $N_o^*$ | $t_o$ |
| Mosaic Detection System | $S_2^*$ | $N_2^*$ | $t$ |
| Frequency Coded System | $S_n$ | $N_n$ | $t$ |
| Orthogonal Detection System | $S_3^*$ | $N_3^*$ | $t$ |

| | |
|---|---|
| $n$ | Number of Resolution Elements |
| $t$ | Frame Time |
| $F$ | Scanning Function |
| $\varphi$ | Focal Length |
| $D$ | Aperture Diameter |
| $R$ | Range of Object |
| $a_2$ | Linear Size of Object |
| $a_1$ | Linear Size of Image |
| $a_o$ | Linear Size of Photocell |
| $T_2$ | Absolute Temperature of Object |
| $T_1$ | Absolute Temperature of Image |
| $F_2$ | Flux Density of Object |
| $F_1$ | Flux Density of Image |
| $E_2$ | Total Flux of Object |
| $E_o$ | Total Flux on Aperture |
| $\epsilon$ | Refractive Index of Immersed System |
| $\sigma$ | Stefan Boltzman Constant |

APPENDIX

THE DESIGN OF INFRARED IMAGING SYSTEMS

## I.  INTRODUCTION

The main body of this report illustrates the desirability of making the sensitive area for the photocell as small as practical. It also analyzes the signal-to-noise ratio of various imaging systems. It is desirable to utilize as much integration time as possible to enhance signal-to-noise ratio. If a mosaic system were designed where each resolution element of the image has its own sensor, the full frame time could be used as integration time, thus obtaining optimum signal-to-noise ratio. Such a system requires many sensors and associated amplifiers to obtain high resolution and wide angle views and is therefore, in many cases, impractical due to complexity, weight, power-consumption and cost. It is therefore desirable to multiplex as many resolution elements as feasible unto one sensor. This multiplexing can be achieved by timesharing. However, timesharing reduces the integration time available and thereby, the signal-to-noise ratio. Another method of multi-plexing can be used by coding each resolution element with a different frequency. This method is advantageous in cases where the response time of the cell is fast, compared to the frame time available, in which case the frequency coded system can approach the signal-to-noise ratio of the mosaic system.

The previous part restricts itself to theoretical analysis and does not treat the practical design problems of dissection of two-dimensional images or the methods of image reconstruction. This appendix will give some practical design examples based on the previous analysis.

## II.  TWO-DIMENSIONAL IMAGE DISSECTION

Optical systems usually project two-dimensional images. Chopping reticles such as the one shown in Figure 5 are suitable to process one-dimensional arrays of resolution elements. For example, the reticle shown in Figure 5 contains a different chopping frequency on each radius.
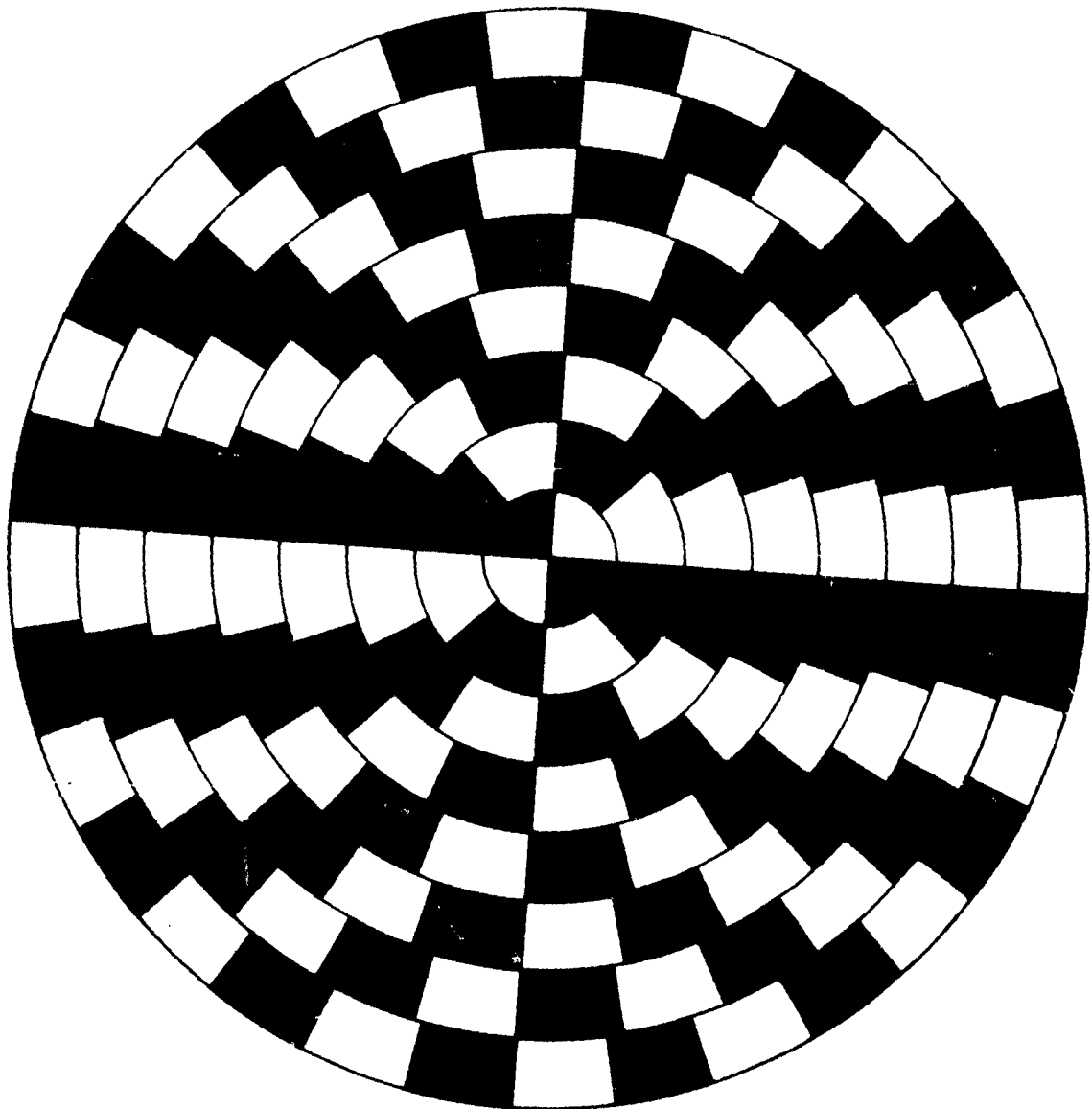
f = 4, 8, 12, 16, 20, 24, 28 and 32 CYCLES/REV

Figure 5. Frequency Coded Reticle.

There is no difference in signal output if the resolution element is at another angular position as long as it is at the same radius. Note the reticle will cut off approximately one-half of the radiant energy.

### A. Modulation of Carrier

A reticle, as shown in Figure 6, has a square wave amplitude modulation superimposed on the carrier modulation of Figure 5. With this amplitude modulation, a light spot in the image field will have a phase as well as a frequency. If the phase relationship of the reticle is known, the angular position of the light spot can be determined. The system requires the transmission of an electrical signal indicating the position of the reticle. Since the signals repeat in one revolution of the reticle, two such electrical signals, preferably at right angle to each other for unambiguous determination of target position, is required. Note the reticle cuts off approximately one-fourth of the radiation intensity in the image. Furthermore, it is necessary to extract phase and frequency of the signal. If there is more than one signal at one radius of the reticle, the extraction of phase information becomes extremely difficult or impossible. Only the stronger signal or the phase of the center of gravity of the signals present may be obtained. Although this modulation of the carrier signal can be somewhat improved by going from amplitude modulation shown to a frequency modulation, the entire scheme is only applicable to tracking systems where one must deal with one light-point only. For imaging systems required for surveillance and search, the modulation of the carrier is not suitable and other means must be found to dissect two-dimensional images.

### B. Convert Two-Dimensional Image Into a One-Dimensional Array

Inability to process two-dimensional images directly, they must be converted into one-dimensional linear arrays. There are three basic ways this can be accomplished, namely, optically, by timesharing, or by use of multiple sensors. These three ways are discussed in greater detail in the following paragraphs.
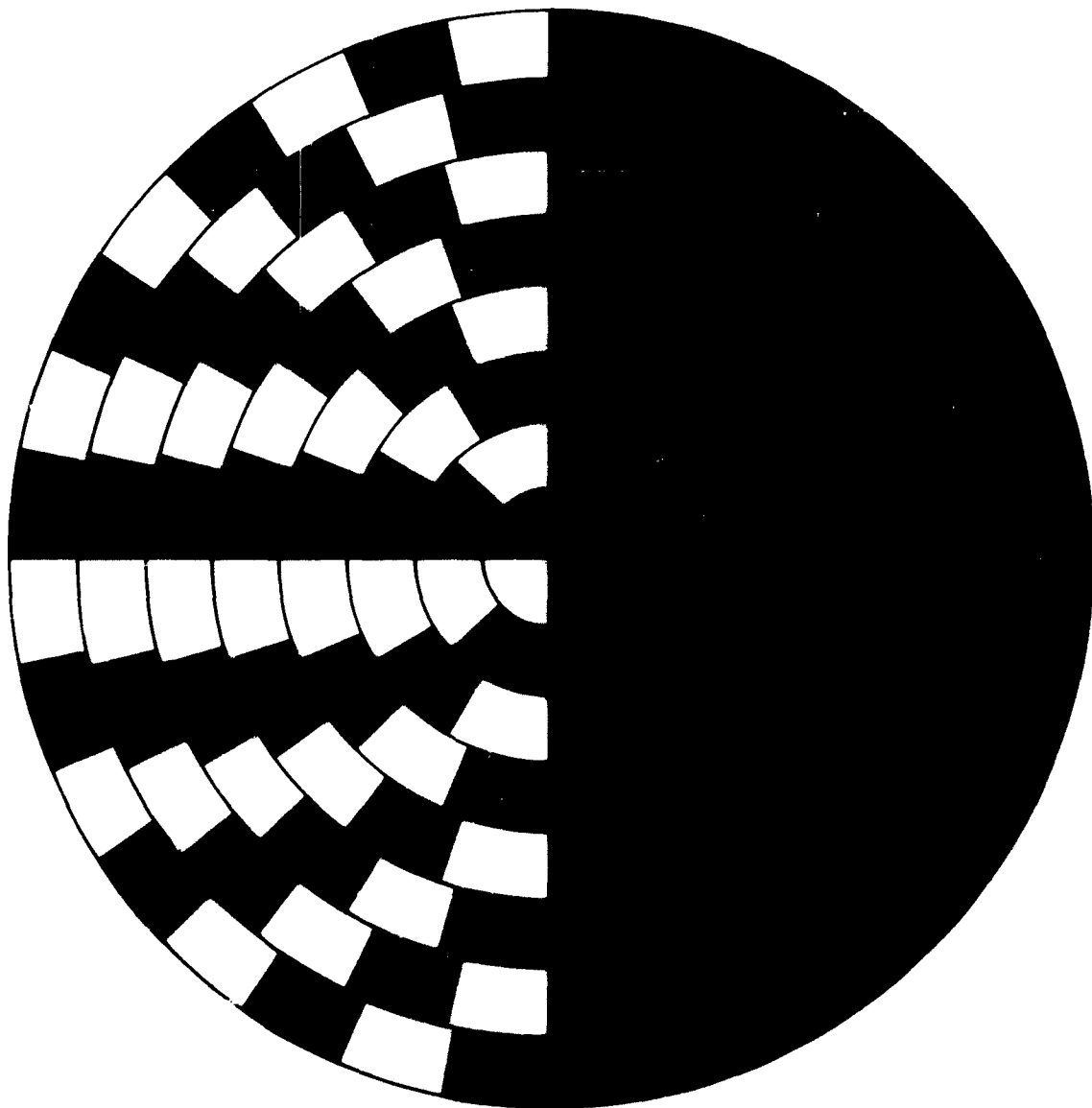
PHASE OF SIGNAL DETERMINES ANGULAR POSITION

f = 4, 8, 12, 16, 20, 24, 28 and 32  CYCLES / REV

Figure 6.   Frequency Coded Reticle
(Amplitude Modulated).

1. <u>Optical Conversion</u>

Using fibre optics, it is possible to rearrange a two-dimensional array of resolution elements into a linear one-dimensional array, which then can be dissected with a reticle similar to the one shown in Figure 5.

In most cases of practical interest, there are too many resolution elements to make it practical to process them all with one reticle, or with one sensor. Therefore, this method, even if the difficulties of infrared fibre optics are resolved, is only of academic interest.

2. <u>Timesharing</u>

Timesharing or sequential scanning results in a reduction on integration time available. Figure 7 shows a disc which can be used in conjunction with the reticle of Figure 5 for this purpose. The disc of Figure 5 has one revolution per frame time, while the reticle of Figure 5 rotates extremely fast. Its speed of rotation is so fast that the highest frequency of the outer ring is equal to one-half the response time of the sensor used. Note the integration time of this system is $a/360$ of the integration time of the previous system. (See Figure 7, $a$ in degrees.) The angular position of the disc of Figure 7 must be known to determine angular position of light spots. This system is less complex but has a reduced sensitivity due to the reduced integration time. Furthermore, note that the frequency coded reticle is only workable if the integration time available is large compared to the response time of the sensor. (See Eq. 33.) This fact limits the usefulness of the time-sharing method severely unless sensors with faster time constants become available.

3. <u>Use of Multiple Sensors</u>

The two-dimensional image is dissected into horizontal strips as shown in Figure 8. Each horizontal strip is processed by one sensing cell. Using a reticle as shown in Figure 9, moving vertically, the linear array of horizontal resolution elements is frequency coded. Note in this system, one has frequency coding in the horizontal direction and an array-like use of sensors in the vertical direction. A linearly moving belt reticle
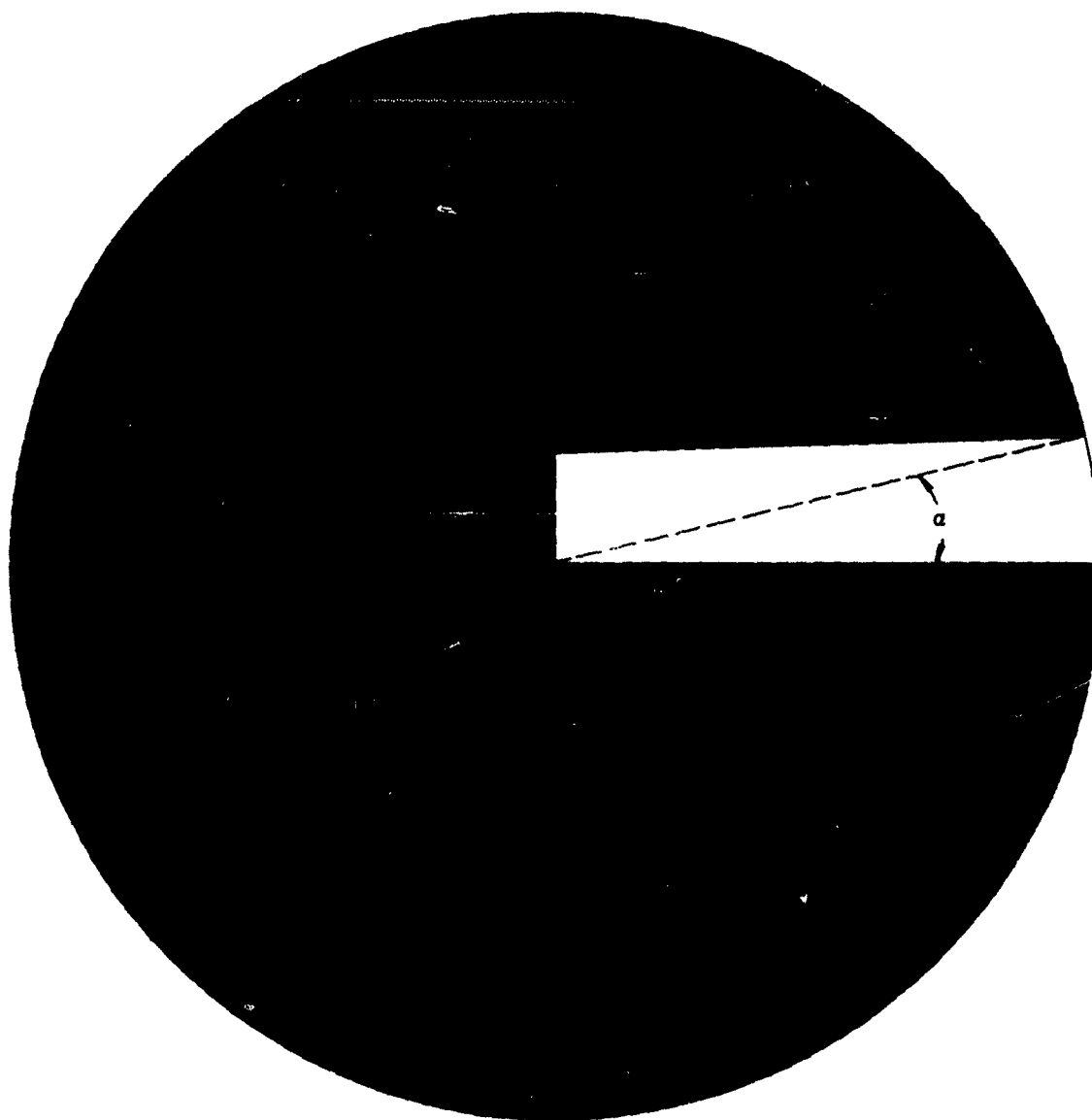
Figure 7.  Timesharing Disc - 1 Revolution/Frame Time
(To be Combined with Reticle of Figure 5).
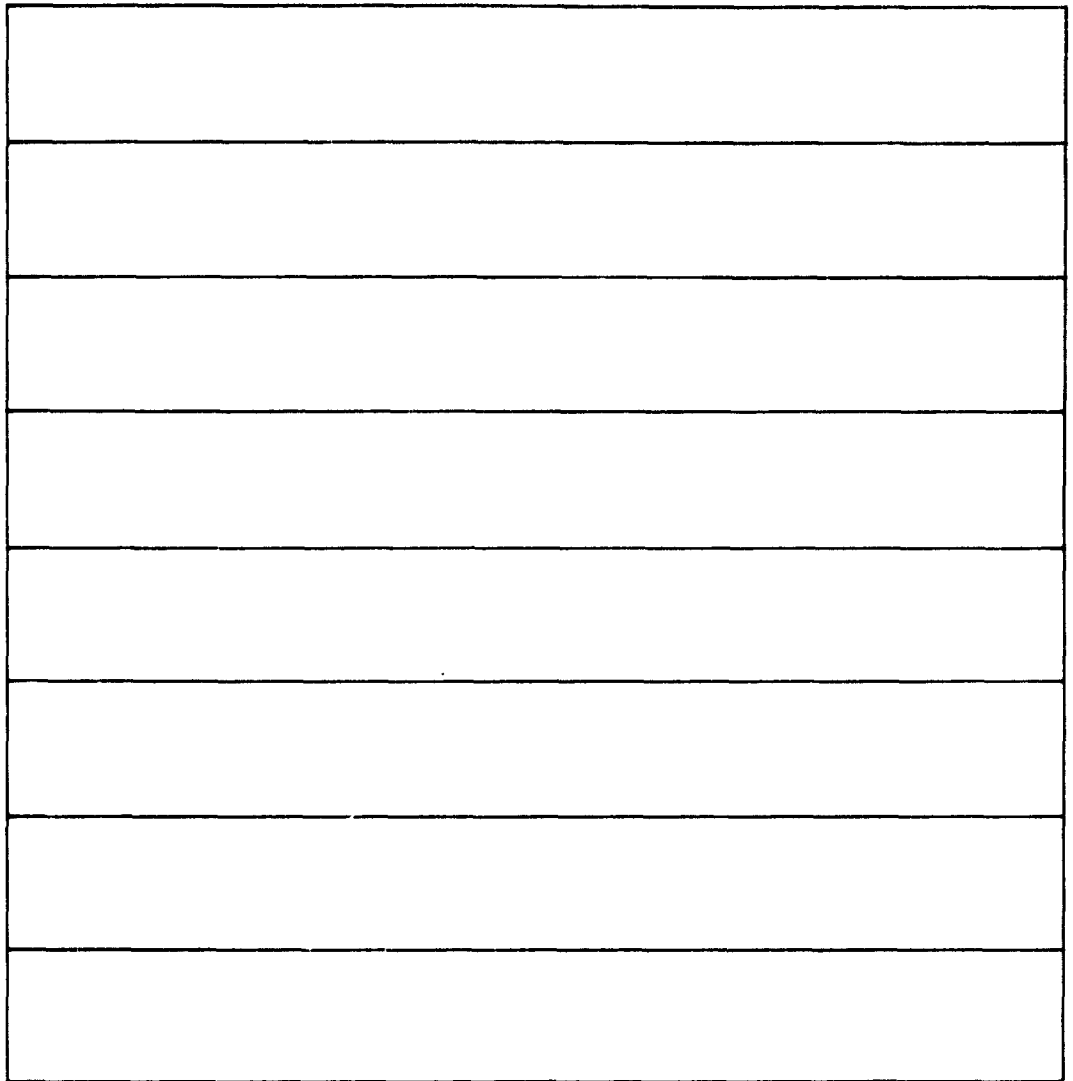
Figure 8.   Image Area Divided into 8 Horizontal Strips.
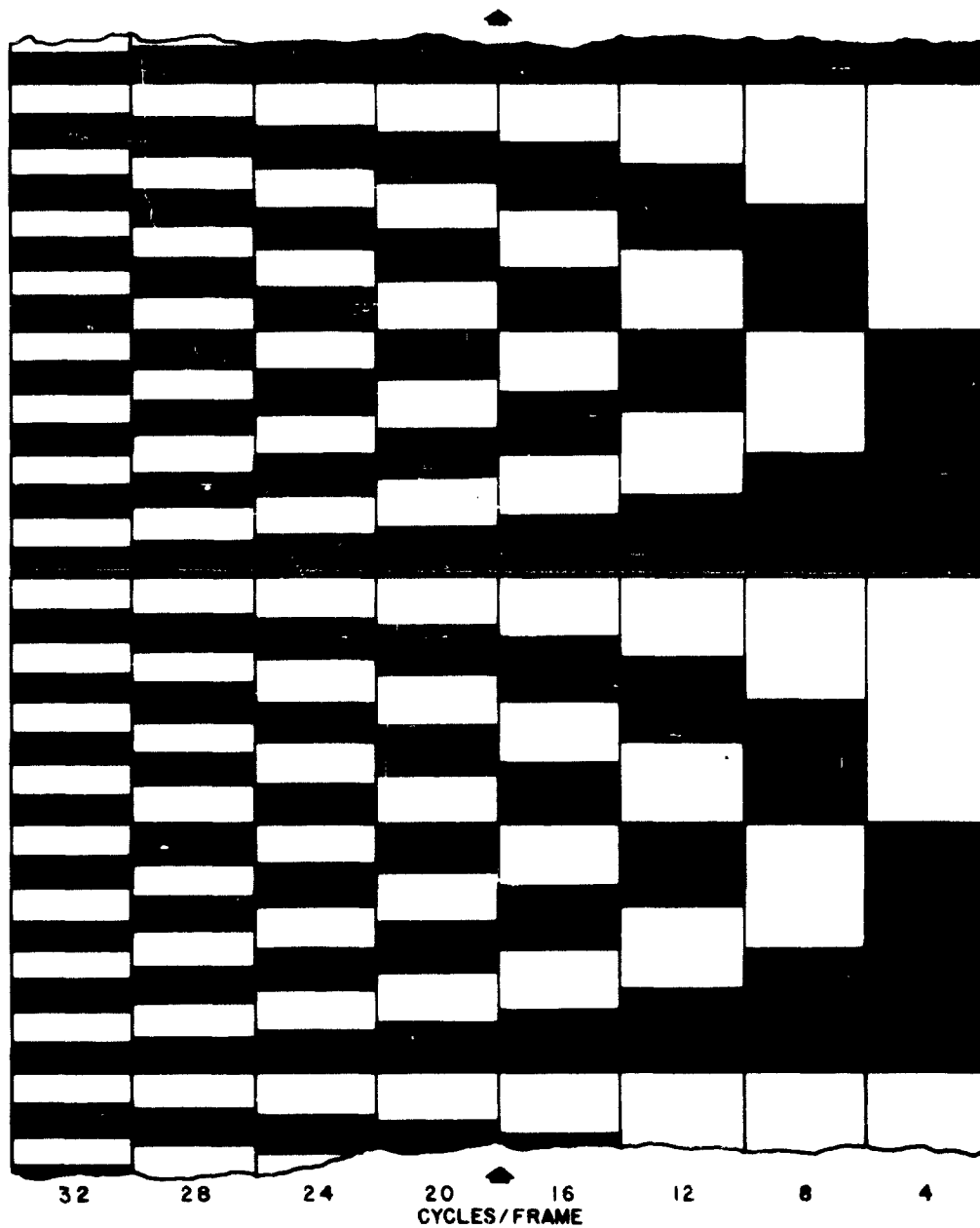
Figure 9. Reticle Belt Moving Vertical.

has been chosen to achieve more even division of the image area into individual resolution elements, as would be possible with a rotating circular reticle. All the light of one horizontal strip has to be collected by a lens or an optical wave guide unto a small sensor cell area. For images having many resolution elements, this method using a combination of many frequencies and multiple sensors is very suitable.

C. Comparison of Imaging Systems Discussed

Of the various possibilities to design imaging systems using frequency coded reticles, the following conclusions can be drawn.

1. Modulation of the carrier is not suitable for imaging systems. It is only useful for tracking systems.

2. A comparison table of two-dimensional imaging systems which use frequency coding in one dimension follows.

| System | 1 | 2 | 3 |
|---|---|---|---|
| 1st dimension | frequency coded | frequency coded | frequency coded |
| 2nd dimension | frequency coded | timeshared | multiple sensors |
| Remarks | complex optics | simple | many amplifiers |
| Sensors required | fast response for many resolution elements | extremely fast time constant necessary | available |
| Signal/noise | good | poor | best |

Most practical applications will use System 2 or 3 or a combination thereof.

### D. Numerical Design Example

We choose, as a numerical example, a square image having

100 resolution elements on one side

10,000 resolution elements total

1 second frame time

$\tau$ = 1 millisecond time constant of sensor

The signal-to-noise ratio of the system is determined by the integration time available.

### 1. Comparison System

The comparison system is a sequential scanning system examining each resolution element in a time sequence. Such a system would therefore have only 1/10,000 of a second available for 1 resolution element. It would require a sensor with a faster time constant. Since this sequential scanning system is used only for purposes of comparison, this fact is disregarded. Its signal-to-noise ratio is called So/No .

### 2. System Number 2

| | | |
|---|---|---|
| Vertical: | Timesharing | 100 resolution elements |
| Horizontal: | Frequency coding | $n_2$ = 100 resolution elements |
| Integration time available | | $t_2$ = 1/100 second |

According to Equation (33)

$$n_2 \frac{\tau}{t_2} \quad \text{must be small compared to 1}$$

For this case:

$$n_2 \frac{\tau}{t_2} = 100 \frac{1/1000}{1/100} \quad \text{and}$$

therefore this condition is not met and this system cannot be used.

3. System Number 3

| | | |
|---|---|---|
| Vertical: | Multiple sensor | 100 resolution elements |
| Horizontal: | Frequency coding | $n_3$ = 100 resolution elements |
| Integration time available | | $t_3$ = 1 second |

According to Equation (33)

$$n_3 \frac{\tau}{t_3} = 100 \frac{1/1000}{1} = \frac{1}{10} \quad \text{which is small compared to 1}$$

therefore this system is feasible.

These evaluations show that the use of frequency coded reticles combined with multiple sensors can result in considerable improvement of signal-to-noise ratio. The system described requires 100 sensors with associated amplifiers.

Note: To apply the same principles to imaging systems with 1 million resolution elements would require sensors with faster time constants in order to be able to process, in the given frame time, more resolution elements with the same sensing cell and arrive at a reasonable total number of sensors.

E. Conclusions on Image Dissection Methods

The frequency coded reticle combined with multiple sensors offers suitable means to design imaging systems with high signal-to-noise ratio. For systems with many resolution elements, it is necessary to operate with low frame rates and therefore, becomes desirable to use faster responding sensing cells. Indium-arsenide sensors may be suitable for such applications. Suitable impedance matching devices to the required amplifiers have to be used with this low impedance sensor.

### D. Numerical Design Example

We choose, as a numerical example, a square image having

100 resolution elements on one side

10, 000 resolution elements total

1 second frame time

$\tau$ = 1 millisecond time constant of sensor

The signal-to-noise ratio of the system is determined by the integration time available.

#### 1. Comparison System

The comparison system is a sequential scanning system examining each resolution element in a time sequence. Such a system would therefore have only 1/10, 000 of a second available for 1 resolution element. It would require a sensor with a faster time constant. Since this sequential scanning system is used only for purposes of comparison, this fact is disregarded. Its signal-to-noise ratio is called So/No .

#### 2. System Number 2

| Vertical: | Timesharing | | 100 resolution elements |
|---|---|---|---|
| Horizontal: | Frequency coding | $n_2$ = | 100 resolution elements |
| Integration time available | | $t_2$ = | 1/100 second |

According to Equation (33)

$$n_2 \frac{\tau}{t_2}$$ must be small compared to 1

For this case:

$$n_2 \frac{\tau}{t_2} = 100 \frac{1/1000}{1/100}$$ and

therefore this condition is not met and this system cannot be used.

     c.    Light is transduced into an electrical square wave.

     d.    Electrical square wave is biased to have positive and negative values.

     e.    Electrical wave is used to modulate an illuminated electric lamp so as to increase and decrease the existing illumination.

     The illumination of the electric lamp, before it is modulated, should be adjusted in order for the maximum negative modulation amplitude to produce approximately zero light output.

     For example, where the frequencies of modulation are 1, 2, 4, 6, 8, and 10 cycles per frame time, Figure 10 shows the modulated light output. Using a reticle exactly like the dissecting reticle and synchronized with the dissecting reticle, the light from the electric lamp diffused into a linear array behind this reticle will show in the corresponding resolution element where it was originally in the image. The mathematical operation performed by this filtering or image reconstruction reticle is,

$$A_i = \sum_{\substack{l=1 \\ j=1 \\ t=0}}^{\substack{t=t_0 \\ j=n \\ l=n}} \int_{t=\frac{U}{2f_1}}^{t=\frac{V}{2f_1}} A_j \, dt$$

where

$t_o$    frame time

n    number of resolution elements

f    chopping frequency

u    even integer

v    odd integer

This operation is illustrated in Figure 10.

It will be difficult to devise a more compact and efficient filter than this optical correlation through a second reticle described.

## IV. SYSTEM DESCRIPTION

An example of such a system using moving belt reticles is illustrated in Figure 11. This system uses multiple sensors and multiple glow lamps in the vertical direction and multiple frequencies in the horizontal direction. The reconstructed visible image can be viewed directly or further processed using standard television techniques.

## V. RECOMMENDATION

It is recommended that tests on the described system be conducted to evaluate obtainable signal-to-noise ratios experimentally. Furthermore, the development of faster time constant sensors and their practical application is necessary in order to extend this technique to high resolution systems.

$$f_{i+1} = f_i - \frac{2}{t_0}$$

NOTE ORTHOGONALITY:

$$A_i = \sum_{\substack{i=1, j=1 \\ t=0}}^{\substack{t=t_0 \\ i=n, j=n}} \int_{t=\frac{u}{2f_i}}^{t=\frac{v}{2f_i}} A_j \, dt$$

U EVEN INTEGER
V ODD INTEGER

$t_0$ FRAME TIME
$t$ TIME
$f$ FREQUENCY
$A$ AMPLITUDE
$n$ DIFFERENT FREQUENCIES
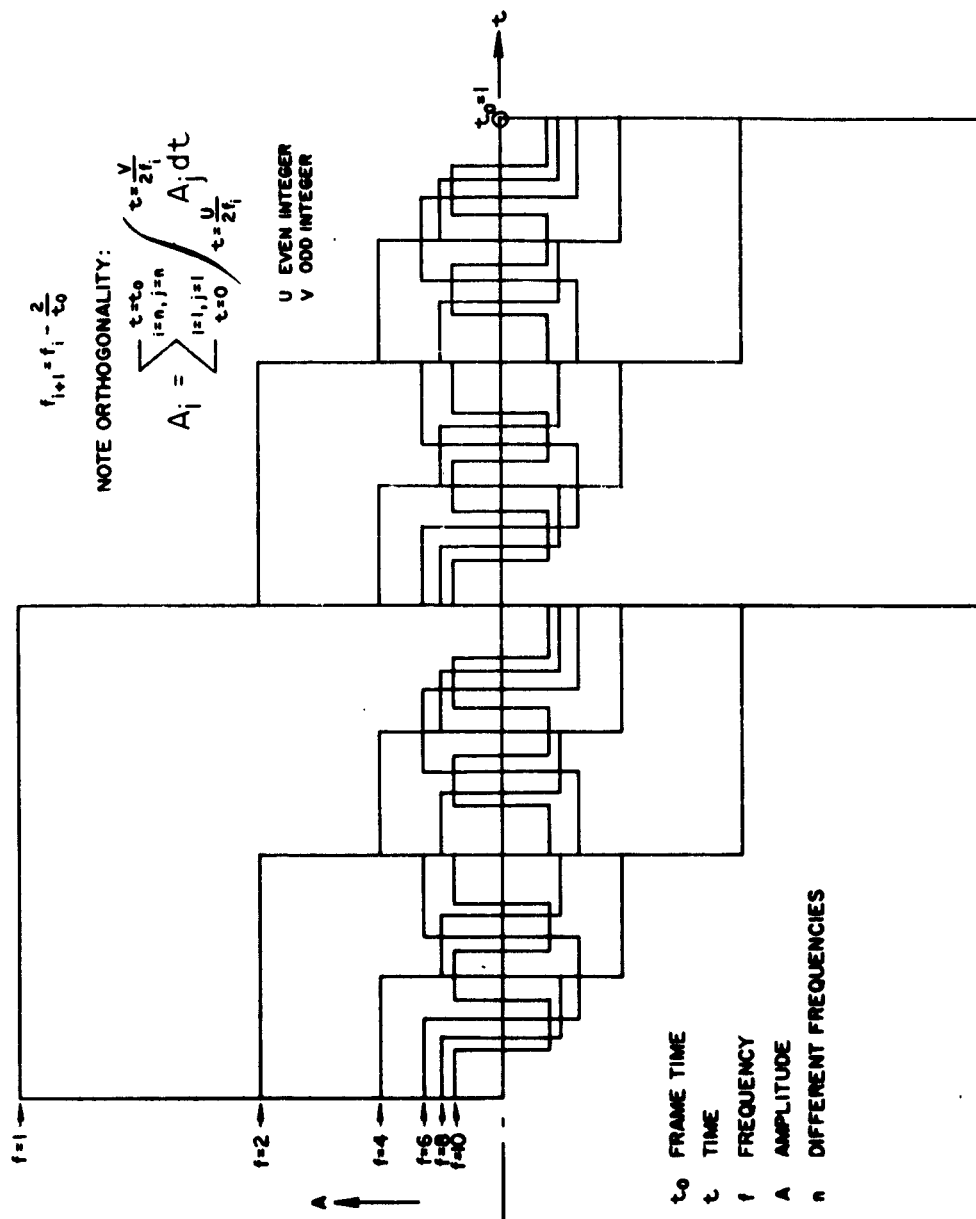
Figure 10. Action of Frequency Filter.
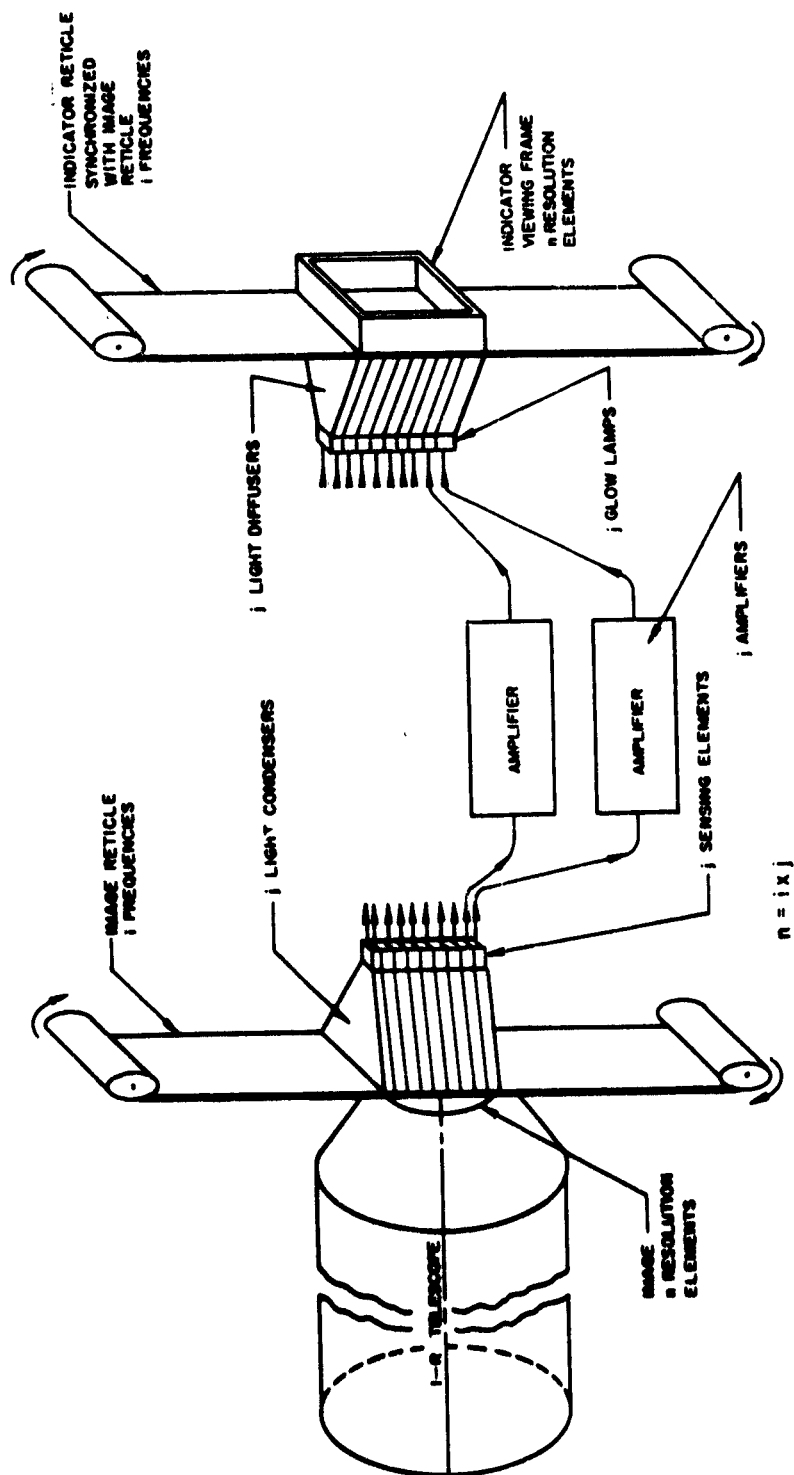
Figure 11.  Example of Reticle Scanning Using Optical
Correlation in Indicator.

Aerospace Corporation, El Segundo, California
ANALYSIS OF INTRINSIC PHOTOCONDUCTORS
USED FOR INFRARED IMAGING PURPOSES by
A. S. Gutman, 30 November 1962. 39 pages includ-
ing illustrations. (Report TDR-169(3128-02)TN-1,
SSD-TDR-62-132) Contract No. AF04(695)-169.

Unclassified Report

Infrared imaging systems have been proposed in
which one photocell can detect many different reso-
lution elements of the image by novel means of
scanning. In previous systems one photocell
processes the information of one resolution element
at any one instant of time. This paper attempts to
analyze and evaluate different systems.

Part I - The discussion begins with an analysis of
a photoconductive cell. The reasons for the choice
of photoconductive cells for infrared are discussed.
The signal-to-noise ratio is derived for weak sig-
nals under the assumption that the current noise is
(over)

UNCLASSIFIED

predominant. Changes in cell dimensions and illumination patterns are investigated.

Part II - The cell analysis is applied to imaging systems. Sequential scanning systems, mosaic systems, and reticle scanning with orthogonal functions are treated.

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